z-scores

Adapted from

Statistics for the Behavioral Sciences

Gravetter and Wallnau

z-Scores and Location

- By itself, a raw score or X value provides very little information about how that particular score compares with other values in the distribution.
- A score of X = 53, for example, may be a relatively low score, or an average score, or an extremely high score depending on the mean and standard deviation for the distribution from which the score was obtained.
- If the raw score is transformed into a z-score, however, the value of the z-score tells exactly where the score is located relative to all the other scores in the distribution.

z-Scores and Location (cont.)

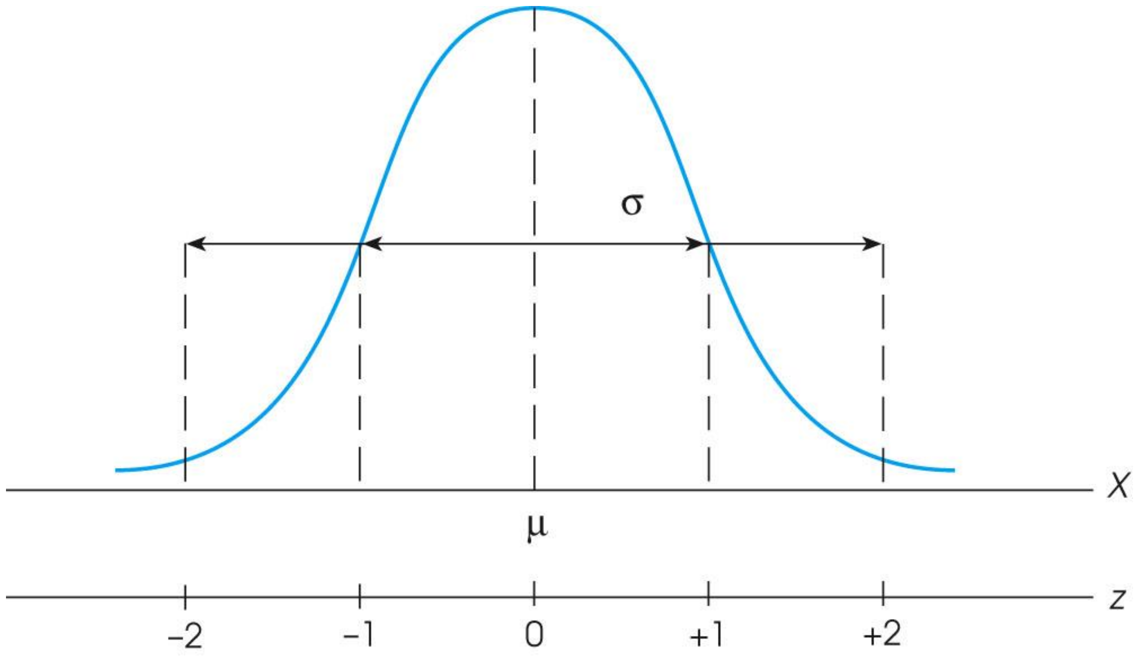
- The process of changing an X value into a z-score involves creating a signed number, called a **z-score**, such that

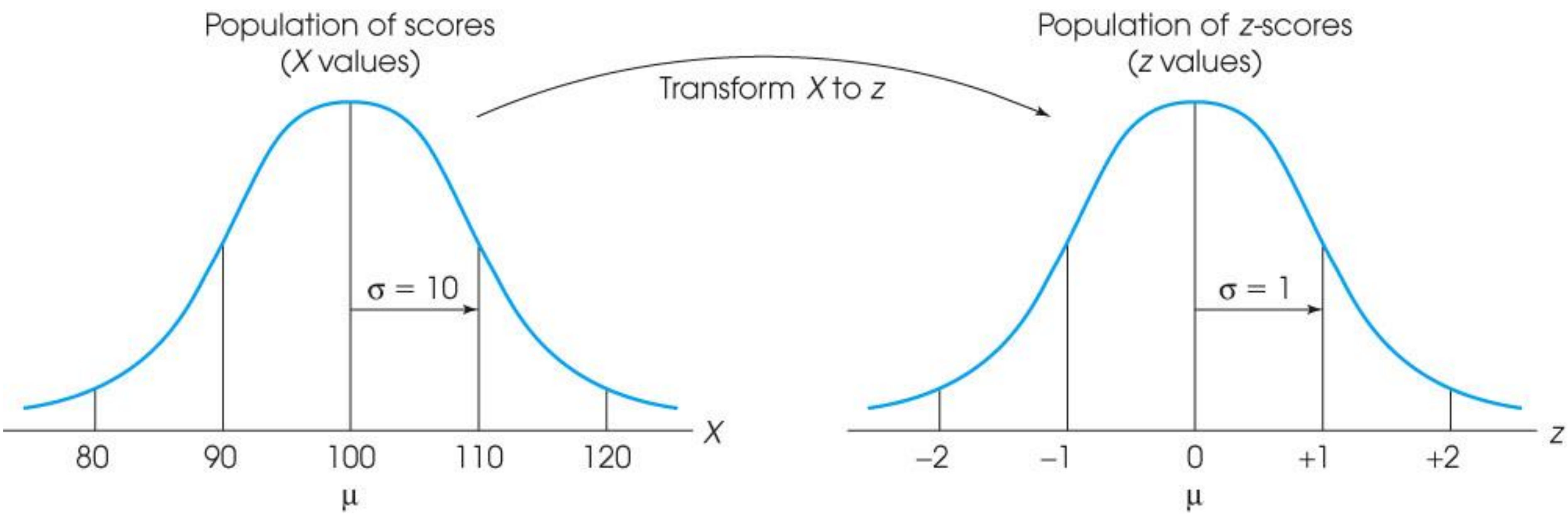
  a. The sign of the z-score (+ or –) identifies whether the X value is located above the mean (positive) or below the mean (negative).

  b. The numerical value of the z-score corresponds to the number of standard deviations between X and the mean of the distribution.

z-Scores and Location (cont.)

- Thus, a score that is located two standard deviations above the mean will have a z-score of +2.00. And, a z-score of +2.00 always indicates a location above the mean by two standard deviations.

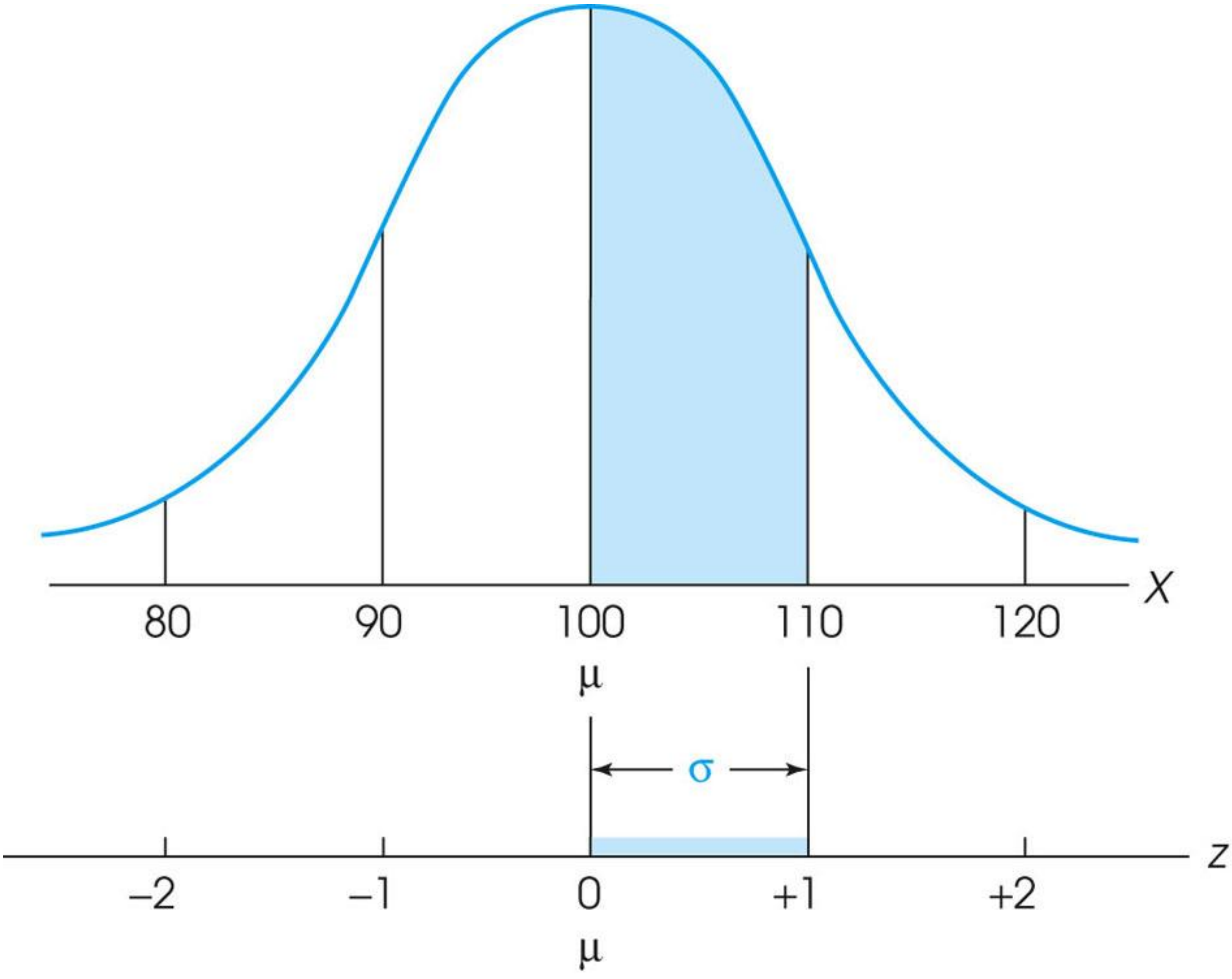

σ
X
μ
z
–2
–1
0
+1
+2


Population of scores
(X values)
Transform X to z
Population of z-scores
(z values)
σ = 10
X
80
90
100
110
120
μ
σ = 1
z
–2
–1
0
+1
+2
μ

Transforming back and forth between X and z

- The basic z-score definition is usually sufficient to complete most z-score transformations. However, the definition can be written in mathematical notation to create a formula for computing the z-score for any value of X.

$$z = \frac{X - \mu}{\sigma}$$


X
80
90
100
110
120
μ
σ
z
–2
–1
0
+1
+2
μ

Z-scores and Locations (cont.)

- The fact that z-scores identify exact locations within a distribution means that z-scores can be used as descriptive statistics and as inferential statistics.
  - As descriptive statistics, z-scores describe exactly where each individual is located.
  - As inferential statistics, z-scores determine whether a specific sample is representative of its population, or is extreme and unrepresentative.

z-Scores as a Standardized Distribution

- When an entire distribution of X values is transformed into z-scores, the resulting distribution of z-scores will always have a mean of zero and a standard deviation of one.
- The transformation does not change the shape of the original distribution and it does not change the location of any individual score relative to others in the distribution.

z-Scores as a Standardized Distribution (cont.)

- The advantage of standardizing distributions is that two (or more) different distributions can be made the same.
  - For example, one distribution has $\mu = 100$ and $\sigma = 10$, and another distribution has $\mu = 40$ and $\sigma = 6$.
  - When these distribution are transformed to z-scores, both will have $\mu = 0$ and $\sigma = 1$.

z-Scores as a Standardized Distribution (cont.)

- Because z-score distributions all have the same mean and standard deviation, individual scores from different distributions can be directly compared.
- A z-score of +1.00 specifies the same location in all z-score distributions.

z-Scores and Samples

- It is also possible to calculate z-scores for samples.
- The definition of a z-score is the same for either a sample or a population, and the formulas are also the same except that the sample mean and standard deviation are used in place of the population mean and standard deviation.

z-Scores and Samples (cont.)

- Thus, for a score from a sample,

$$z = \frac{X - M}{s}$$

- Using z-scores to standardize a sample also has the same effect as standardizing a population.
- Specifically, the mean of the z-scores will be zero and the standard deviation of the z-scores will be equal to 1.00 provided the standard deviation is computed using the sample formula (dividing n – 1 instead of n).